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Abstract

This paper specifies and solves a two-stage, game theoretic model of a mixed market for crime control. In the first stage of the model, private targets and the government choose levels of policing. In the second stage, criminals choose targets and the severity of the crimes that they commit.

The paper's key results are as follows. First, private policing can both divert crime to targets that lack private protection and also increase the severity of the crime that these less protected targets suffer. Second, an increase in private policing reduces the aggregate expenditure on traditional policing. This is an instance of a political incentive externality, where private policing affects the objective function of the government. Specifically, it reduces the level of traditional policing that is consistent with the Samuelson condition for efficient provision of a public good. Third, the substitution of private for public policing carries with it a change in the technology of policing. In effect, private policing leads to a shift from enforcement and punishment towards monitoring and target hardening. This, in turn, may lead to an increase in the severity of crime. Fourth, the mixed policing equilibrium is inefficient, and in some situations mixing may reduce the utility of all targets.
I. Introduction

Privatization is often analyzed as a binary choice between whether a good should be provided publicly or privately. It is important to remember, however, that there are many shades between the primary colors of completely private and completely public. It is in this part of the public-private spectrum that one finds mixed markets, where both the public and private sectors are active in the provision of some good or service. This definition is quite broad, including private involvement by individual agents supplementing or substituting for public provision and collective organizations that are in some way less public than traditional governments. The latter have become known as private governments, a category that includes both the business improvement districts that provide services to retailers and the residential community associations who provide neighborhood amenities to homeowners. Thus, political disintegration sometimes takes the form of privatization.

Mixed markets are increasingly important. The best estimates are that in the US there were 900 business improvement districts in 1992 (Pack (1992)) and 150,000 residential community associations in 1993, the latter housing a total of 32 million residents (McKenzie (1996)). These organizations create mixed markets for local public services, including crime protection. In addition, the markets for education and transportation are also mixed, with private schools and individual vehicles serving as alternatives to public schools and mass transit.

In this paper, we focus on mixed markets and crime. We do this for several reasons. First, crime is important. Even though the level of crime in the U.S. has stabilized in recent years, public opinion surveys consistently place crime at or near the top of the list of social ills (DiIulio (1996)), and crime levels remain high by historical standards. Estimates of the costs of crime are staggering. Anderson (1999) estimates the total cost of crime in the U.S. to be on the order of $1.1 trillion per year, including $397 billion in crime related production costs (including items such as target hardening, policing and security, and crime induced health care costs), $130 billion in time opportunity costs (including time spent in preventative activities, time lost to the victims of crime, and the opportunity costs of incarceration), and $574 billion in costs for lost life and health.

Second, crime control is a heavily and increasingly mixed market. Both public and private agents and institutions play important roles in the provision of policing services. Narrowly defined, the main participants in the mixed market for crime control are uniformed public police officers on the one hand, and private investigators and
private security guards on the other. More broadly conceived, public sector participants could include all employees of the criminal justice and penal systems, while private sector participants could include locksmiths, workers involved in the manufacture, installation and monitoring of private security devices, telephone and computer system security consultants, and many other security related occupations.

Surprisingly, it appears that private security activities are much larger than public security activities in most countries (Sklansky (1999, p. 1181)). For example, Cunningham et al (1990, pp. 228-229) report that expenditures on private security were 1.7 times as large as expenditures on public law enforcement in the U.S. in 1990, and that there were roughly 2.4 private security employees for every law enforcement employee in the U.S. at that time. A 1989 survey of private and public policing in California, Indiana, Michigan, Texas and Missouri indicates that the ratio of private to public employment in crime control was 3.3. Campbell and Reingold (1994) report that in 1991 121,000 persons were employed as private security guards and private investigators in Canada compared to 57,000 uniformed police officers, so the ratio of private to public employment in policing (narrowly defined) was about 2.1 in this case.

Furthermore, the gap between private security and public policing is growing. Cunningham et al (1990, Tables 7.12 and 7.13) report that expenditures on private security activities increased from $3.5 billion to $52 billion between 1970 and 1990, while expenditures on public law enforcement rose from $6 to $30 billion over this period. Thus, during this time period in the U.S., expenditures on private security grew more than 3 times as fast as expenditures on public law enforcement. Similarly, Campbell and Reingold [1994] note that during the 1971 to 1991 period, the number of public police officers in Canada increased by 41%, while the number of private investigators increased by 71% and the number of private security guards increased by 126%. The population of Canada increased by about 30% over this time period.

This paper specifies and solves a two-stage, game theoretic model of a mixed market for crime control. In the first stage, private targets and the governments choose levels of policing. In the second stage, criminals choose targets and the severity of the crimes that they commit. The micro model of crime in the second stage of the game is important. Much analysis of the provision of local public goods supposes that consumers derive utility from an abstract g. This paper will show that in understanding a mixed market, it may be necessary to specify a model that respects the microstructure of the good in question. Our results on mixing are tightly related to our model of crime.

The paper reaches several interesting conclusions. First, private policing can both divert crime to targets that lack private protection and also increase the severity of the
crime that these less protected targets suffer. Second, an increase in private policing reduces the aggregate expenditure on traditional policing. This is an instance of a political incentive externality, where private policing affects the objective function of the government. Specifically, it reduces the level of traditional policing that is consistent with the Samuelson condition for efficient provision of a public good. Third, the substitution of private for public policing carries with it a change in the technology of policing. In effect, private policing leads to a shift from enforcement and punishment towards monitoring and target hardening. This, in turn, may lead to an increase in the severity of crime. Fourth, the mixed policing equilibrium is inefficient, and in some situations mixing may reduce the utility of all targets. Taken together, these results suggest that the growth of private policing may have some undesirable consequences. In the conclusion, we will discuss policy alternatives that address these consequences.

This paper builds on three strands of the public economics literature. First, it builds on the literature on the economics of crime. This literature, beginning of course with Becker (1974), has considered the responses of criminals to punishment. Hui-Wen and Png (1994) consider private policing and show the possibility of crime diversion, while Helsley and Strange (1999) consider the formation of gated communities, showing that private policing typically involves both diversion and deterrence. None of these papers have considered the severity of crime or the differences in policing technology between private and public providers. The second literature related to our paper concerns the private provision of public goods. Epple and Romano (1996) consider an individual private supplement to a public good. Helsley and Strange (1998, 2000b) consider the formation of so-called private governments. These largely voluntary organizations provide group supplements to publicly provided goods. Among the most important instances of private governments are residential community associations, gated communities, and business improvement districts. The third literature that is related to our paper deals with decentralization. This topic is, of course, central to the efficiency of the economics of political integration and disintegration. See Scotchmer (2001) for a recent survey. Our results on the inefficiency of the mixed market equilibrium are related to recent work analyzing the kinds of externalities that exist in a decentralized local public economy (Benabou (1993), Helsley and Strange (2000a)). This paper advances the last two literatures by embedding private provision in a model with an explicit treatment of crime. As will be seen below, the analysis of private provision and decentralization depends strongly on the microstructure of the market in question.

The rest of the paper is organized as follows. Section II presents and solves the model of crime. Section III embeds this in a game-theoretic model of a mixed market.
Section IV carries out a normative analysis of a mixed market. Section V discusses extensions of the analysis and policy implications.

II. A model of crime and policing in a mixed market

This section presents a model of crime with both public and private policing. In this section, policing is exogenous. The next section considers the equilibrium determination of public and private policing, using this section's crime model as the second stage of a two-stage game.

A. Targets and instruments of crime control

There are two types of agents, targets and criminals. Targets are differentiated, and \( q_i \) \((q_{i}, q_{i}), i = 1,2,...I,\) represents the value of target \( i.\) A target with a high \( q \) will be more attractive to criminals. Naturally, \( q \) will also influence the losses from crime, with higher \( q \) targets incurring larger losses, other things being equal. Without loss of generality, the index set is chosen so that \( q_1 \leq q_2 \leq ... \leq q_I.\)

The variable \( g \) represents the level of public policing. We suppose that \( g \) must be uniform across targets. Public policing includes activities like monitoring and target hardening that are general in the sense that they reduce the rewards of all crimes. It also includes activities like investigation, prosecution, and punishment. These are specific in the sense that they can reduce the rewards to more serious crimes to a greater degree. This distinction is important. Because specific policing imposes costs on criminals that vary with the severity of crime, it can influence the severity of crime. This is sometimes referred to as marginal deterrence (Polinsky and Shavell (2000), Mookherjee and Png (1994)). For example, the requirement of mandatory sentences for gun crimes is designed to discourage burglars from carrying guns.

The variable \( g_i \) represents the level of private policing of target \( i.\) Targets choose their own levels of \( g_i,\) and so \( g_i \) may vary across targets. Unlike public policing, the scope of private policing is limited to general activities like monitoring and target hardening. This means that private policing impacts all crimes, regardless of their severity. Thus, an important distinction between public and private policing is that only the former has a marginal deterrent effect. Marginal deterrence refers to the impact of raising the penalty for one crime on the choice between crimes of different severity. By this logic, strict mandatory sentences for armed robbers may encourage thieves to commit nonviolent break-and-enter offences rather than violent armed robberies.
Our model is sufficiently general that targets may be thought of as individuals or communities. In the case where targets are communities, private policing could involve the hiring of security guards, the construction of walls and gates, and the use of video surveillance. In the case where targets are individuals, private policing could involve installing alarms, and reinforcing doors and windows.\footnote{The assumption that private policing has no marginal deterrent effect at all is probably too strong. For instance, a video camera that identifies criminals for prosecution results in a greater sanction for the criminal who commits the more serious crime. However, because it is not targeted, the video camera almost certainly has a smaller effect than would a targeted investigation. All of this paper's analysis would continue to hold as long as private policing had a smaller marginal deterrent effect than did public policing.} The distinction in activities is not as sharp as it might seem. A very rich individual would almost certainly deploy the full range.

B. Criminal choices and payoffs

In our model criminals are assumed to respond rationally to incentives. Criminals make three choices. First, they decide whether or not to participate in crime. Second, they choose targets. $n$ denotes the amount of crime directed toward a particular target. In the case where targets are communities, $n$ should be interpreted as the number of active criminals in a community. In the case where targets are individuals, $n$ should be interpreted as the level of crime that a target expects to experience. Third, criminals choose the severity of the crime that they commit, $s$.

Criminal payoff for target $i$ is given by

$$V_i = v(q_i, n, s) - M(g, g_i) - F(g)s. \tag{II.1}$$

$v$ is the gross payoff to crime. We suppose that $v$ increases in $q$ and $s$ and decreases in $n$. $v$ increases in $s$ because the willingness to damage property or inflict injury on targets is likely to allow for more profitable crime. For example, a home invader is likely to get away with more than a non-violent daytime burglar. $v$ decreases in $n$ because of the congestion of criminal opportunities. We assume that $\partial^2 v / \partial n^2 \geq 0$, $\partial^2 v / \partial s^2 < 0$, $\partial^2 v / \partial n \partial s \geq 0$, $\partial^2 v / \partial q \partial n \geq 0$, $\partial^2 v / \partial q \partial s > 0$, and that the higher order derivatives of $v$ are all zero. The roles of these assumptions will become clear as we go along.

The rest of the criminal utility function is easy to grasp. $M$, the reduction in payoff associated with public and private monitoring, is assumed to be increasing in both arguments. We assume that $\partial^2 M / \partial g^2 \geq 0$ and $\partial^3 M / \partial g^2 \partial g \geq 0$. We also assume $\partial^2 M / \partial g \partial g \partial g \geq 0$, so public and private policing are substitutes in the sense that an increase in either...
instrument reduces the marginal impact of the other. The term $F(g)s$ is the reduction in payoff for severity $s$. We assume that the function $F$ is increasing and concave. A more severe crime is expected to be punished more stringently, with the expectation of punishment increasing in the expenditure on public policing.

C. Severity choice

For any target, $\frac{\partial V}{\partial s} = \frac{\partial v}{\partial s} - F(g) = 0$ implicitly defines the maximizing severity $s_i(q_i,n,g)$. The second-order condition for the severity choice problem, $\frac{\partial^2 v}{\partial s^2} < 0$, which we have assumed to be met.

The comparative statics of $s_i$ are

$$\frac{\partial s_i}{\partial q_i} = -\frac{(\partial^2 v/\partial s \partial q_i) / (\partial^2 v/\partial s^2)} {> 0}, \quad (II.2)$$

$$\frac{\partial s_i}{\partial n} = -\frac{(\partial^2 v/\partial s \partial n) / (\partial^2 v/\partial s^2)} {\geq 0}, \quad (II.3)$$

$$\frac{\partial s_i}{\partial g} = \frac{F'(g) / (\partial^2 v/\partial s^2)} {< 0}. \quad (II.4)$$

The most interesting comparative static here is $\frac{\partial s_i}{\partial n}$, which has the sign of $\frac{\partial^2 v}{\partial s \partial n}$. If $\frac{\partial^2 v}{\partial s \partial n} > 0$, then the severity of crime rises with the level of crime in this model. $\frac{\partial^2 v}{\partial s \partial n} > 0$ means that the marginal reward from a more serious crime is greater the higher is the overall level of crime. For example, in a high-crime neighborhood, there are few easy criminal opportunities left. If there are no valuable cars to steal, then car theft may require the more serious crime of car-jacking.

Is it reasonable to assume $\frac{\partial^2 v}{\partial s \partial n} > 0$? Table B-5 of State and Metropolitan Area Data Book, 1997-1998 (U.S. Department of Commerce, Bureau of the Census) gives reported crime rates (crimes known to police) and the percentage of these that are "violent" (murder, non-negligent manslaughter, forcible rape, robbery, and aggravated assault) by MSA for 1995. The simple correlation between the percentage of crimes that are violent and the crime rate in this data set is +0.28. Regressing the percentage of crimes that are violent on the crime rate (also expressed as a percentage) gives a slope coefficient of +0.67 with a t-statistic of 5.1. Of course, this relationship could a spurious correlation arising from the crime reporting process: in high crime areas, more non-violent crimes may go unreported. (II.4) indicates that, under our assumptions, severity is decreasing in the level of public policing, as expected.
D. Target choice

Criminals choose targets to maximize payoffs. If a positive amount of crime is directed toward every target, then perfect criminal mobility implies that every target must yield the same criminal payoff in equilibrium. Letting $V$ denote this common payoff level, the amount of crime directed toward target $i$ is the value of $n$ that solves

$$V_i = v(q_i, n, s_i) - M(g_i) - F(g)s_i = V. \tag{II.5}$$

Denote the solution to (II.5) for $n$ by $n_i(q_i, g_i, V)$.

Using the first-order condition for $s$, the comparative statics of $n_i$ are:

$$\frac{\partial n_i}{\partial q_i} = \frac{-v}{v/\partial n} > 0, \tag{II.6}$$

$$\frac{\partial n_i}{\partial g_i} = \frac{M/\partial g_i}{v/\partial n} < 0, \tag{II.7}$$

$$\frac{\partial n_i}{\partial g} = \frac{\partial M/\partial g + F'(g)s_i}{v/\partial n} < 0, \tag{II.8}$$

$$\frac{\partial n_i}{\partial V} = \frac{1}{v/\partial n} < 0. \tag{II.9}$$

(II.6) indicates that more valuable targets attract more crime. (II.7) and (II.8) show that increases in private or public policing reduce the amount of crime directed toward target $i$, other things being equal. (II.9) says that the amount of crime directed toward target $i$ decreases as the criminal payoff increases due to congestion.

E. Market clearing

The criminal payoff level $V$ is determined by the choice between crime and other activities. There are two ways to model the determination of $V$. The simplest is to assume that the criminal labor market is open. In this case, there is an infinite supply of potential criminals who are willing to be active as long as they receive some exogenous reservation payoff level, $V^*$. The other possibility is to assume that the criminal labor market is closed, and that the supply curve of crime is upward sloping in the level of the

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2 In order for there to be a positive amount of crime directed toward every target, we need the first few crimes to be very valuable to criminals. $\lim_{x \to 0} v([x], n, s_i) = \bullet$ will suffice. The assumption that all targets are victimized is parallel to the "full market" assumption in markets of product differentiation.
criminal payoff. In this case V* clears the aggregate market for crime in the sense that all criminals earn the same level of utility and there is no incentive to enter or leave the criminal labor market. In this section, we concentrate on the closed specification, letting N^S(V) be the upward sloping supply curve of crime.

The demand for crime gives the amount of criminal activity that can occur with each criminal receiving the payoff level V. Formally, the demand curve for crime is N^D(\mathbf{q}, g, V) = \mathbf{n}_t(\mathbf{q}, g, V), where the bold characters represent the vectors \mathbf{q} = (q_1, q_2, ..., q_I) and \mathbf{g} = (g_1, g_2, ..., g_I). Let N^D = \partial N^D/\partial V, where N^D < 0 by (II.9).

The market clearing criminal payoff level V* satisfies N^D(q, g, V*) - N^S(V*) = 0. This condition implicitly defines V* as a function of the target values, and the levels of private and public policing: V*(\mathbf{q}, g). The comparative statics of V* are

\[ \partial V*/\partial q_i = -\left(\frac{\partial n_i}{\partial q_i}\right)(N^D - N^S) > 0, \]  
\[ \partial V*/\partial g_i = -\left(\frac{\partial n_i}{\partial g_i}\right)(N^D - N^S) < 0, \]  
\[ \partial V*/\partial g = -\left(\frac{\partial n_i}{\partial g}\right)(N^D - N^S) < 0. \]

The market clearing amount of crime for target i is n_i* = n_i(\mathbf{q}, g, V*(\mathbf{q}, g)). The comparative statics of n_i* are

\[ \partial n_i*/\partial q_i = \partial n_i/\partial q_i + (\partial n_i/\partial V)(\partial V*/\partial q_i) \]  
\[ = (\partial n_i/\partial q_i)(\mathbf{n}_j, \partial n_j/\partial V - N^S)/(N^D - N^S) > 0, \]  
\[ \partial n_i*/\partial q_j = \partial n_i/\partial q_j + (\partial n_i/\partial V)(\partial V*/\partial q_j) \]  
\[ = (\partial n_i/\partial q_j)(\mathbf{n}_k, \partial n_k/\partial V - N^S)/(N^D - N^S) < 0, \]  
\[ \partial n_i*/\partial g = \partial n_i/\partial g + (\partial n_i/\partial V)(\partial V*/\partial g) \]  
\[ = [\partial n_i/\partial g)(\partial n_i/\partial V - N^S) - (\partial n_i/\partial V)(\partial n_i/\partial g)]/(N^D - N^S), \]  
\[ \partial n_i*/\partial g_j = (\partial n_i/\partial V)(\partial V*/\partial g_j) > 0 \quad (j \neq i). \]
The policing variables are of greatest interest. (II.14) shows that private policing has a net deterrent effect on crime: an increase in \( g \) decreases the amount crime directed to target \( i \). However, (II.16) demonstrates that private policing has external effects: an increase in private policing by some other target \( j \) increases crime for target \( i \). Thus, one of the impacts of private policing is to divert crime to other targets. It is obvious from (II.16) that in an open model with an exogenous criminal payoff level, private policing has no diversionary effect. It is interesting that the sign of \( \partial n_i^*/\partial g \) in (II.15) is ambiguous in the general case. This means that it when targets are heterogeneous and the criminal labor market is closed, it is possible that an increase in public policing could increase the level of crime for some (relatively uncongested) targets. However, even in that case, aggregate crime must fall as \( g \) rises. If the criminal labor market is open or if targets are homogeneous, then an increase in public policing decreases the amount of crime directed toward every target.

This analysis may be summarized as follows:

Proposition 1 (diversion and deterrence): When the criminal labor market is closed, an increase in private policing by target \( i \) decreases the amount of crime at \( i \) but increases crime for all other targets. Total criminal activity declines.

Proof: The first two claims follow from the comparative statics. To see that total criminal activity declines, observe that

\[
\frac{\partial V^\ast}{\partial g_i} = -\left(\frac{\partial n_i^*/\partial q_i}{\partial n_i^*/\partial q_i} \right) < 0,
\]

which implies that total crime declines since \( N_{Si} > 0 \). QED.

Proposition 1 relies on the assumption that all targets are victimized to some degree, a kind of "full markets" assumption. If some targets were so unattractive to criminals that they were not victimized in equilibrium, then the details of the analysis would change but not the spirit. Crime diversion would still occur among the victims when any one increased its expenditures on private policing. In addition, previously unvictimized targets might become attractive to criminals if the increase in private policing were sufficiently large.

The severity associated with the market clearing criminal payoff \( V^\ast \) is \( s_i^\ast = s_i(q_i, n_i(q_i, g), V^\ast(q_i, g), g) \). The comparative statics of \( s_i^\ast \) are

\[
\begin{align*}
\frac{\partial s_i^\ast}{\partial q_i} &= \frac{\partial s_i}{\partial q_i} + (\frac{\partial s_i}{\partial n_i})(\frac{\partial n_i^*/\partial q_i}) > 0, \\
\frac{\partial s_i^\ast}{\partial g_i} &= (\frac{\partial s_i}{\partial n_i})(\frac{\partial n_i^*/\partial q_i}) \leq 0,
\end{align*}
\]

(II.17) (II.18)
\[
\frac{\partial s_i^*}{\partial g} = \left(\frac{\partial s_i}{\partial n}\right)\left(\frac{\partial n_i^*}{\partial g}\right) + \frac{\partial s_i}{\partial g},
\]

(II.19)

\[
\frac{\partial s_i^*}{\partial \gamma} = \left(\frac{\partial s_i}{\partial n}\right)\left(\frac{\partial n_i^*}{\partial \gamma}\right) \geq 0 \quad (j \neq i).
\]

(II.20)

The signs of these effects obviously depend on the sign of \(\frac{\partial s_i}{\partial n}\). As noted earlier, there is some evidence that seems to suggest that \(\frac{\partial s_i}{\partial n} > 0\). In this case, (II.18) shows that severity decreases with the level of one’s own private policing. However, the diversionary externality discussed above implies that in this case \(\frac{\partial s_i^*}{\partial \gamma} > 0, i \neq j\). When the severity of crime rises with the crime rate, greater private policing by one target both diverts crime to other targets and increases the severity of crime there. The impact of private policing in one jurisdiction on the severity of crime in another is unique to this paper, since no other model of the economics of crime considers both severity and criminal mobility.

This analysis may be summarized as follows.

Proposition 2 (severity): When the criminal labor market is closed and severity rises with the level of crime, an increase in private policing by target \(i\) decreases severity for \(i\) but increases severity for all other targets.

Proof: The two claims follow from the comparative statics. QED.

In sum, this section has shown that private policing can both divert crime to targets that lack private protection and also increase the severity of the crime that these less-protected targets suffer. The next section analyzes endogenous policing, by embedding the crime model of this section in a simple game theoretic model of a mixed market.

### III. Equilibrium public and private policing

This section considers a two-stage game of policing in a mixed market. In the second stage, criminals choose targets and the severity with which they commit crimes, as analyzed above. In the first stage, individual targets and a government simultaneously choose levels of private and public policing, respectively. Choices in the first stage are made in anticipation of how policing impacts criminal behavior. The solution to this game will be a subgame perfect Nash equilibrium. We will describe below why we consider this modeling strategy is appropriate. We will also consider alternatives.
A. Private and public objectives and decisions

Target i’s utility is given by the twice continuously differentiable function $U_i = u(q_i, n, s) + x$, where $x$ is consumption of a numeraire good, and the strictly decreasing subutility function $u(q_i, n, s)$ captures the losses that a target experiences from crime. We assume that $\partial^2 u / \partial n^2 \leq 0$, $\partial^2 u / \partial s^2 \leq 0$, and $\partial^2 u / \partial n \partial s \leq 0$.

We assume that target i incurs a cost of $c(g_i)$ for private policing, and a cost $c(g)$ for public policing, and that these per capita cost functions are increasing and convex. These assumptions regarding the per capita costs of policing are consistent with cost studies (Schwab and Zampelli (1987)). Then, if target i has income $y$, target i’s utility becomes $y_i + u(q_i, n, s) - c(g_i) - c(g_i)$.

Target i chooses $g_i$ to maximize utility taking the choices of other targets and the public sector as given, but anticipating how its choice will impact the level and severity of crime in the second stage of the game. Thus, target i chooses $g_i$ to maximize

$$U_i^* = y_i + u(q_i^*, n_i^*, s_i^*) - c(g_i) - c(g).$$ (III.1)

The public sector chooses $g$ to maximize aggregate welfare, equal to the sum of individual target utilities, taking the various $g_i$ as given, but anticipating how its choices will impact the level and severity of crime in the second stage of the game. Formally, the public sector chooses $g$ to maximize

$$W^* = \sum_i [y_i + u(q_i^*, n_i^*, s_i^*)] - c(g).$$ (III.2)

This objective function ignores the welfare of criminals.

B. Public policing

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\(^3\) This approach is equivalent to assuming that the public sector finances policing through an equal-share lump-sum tax that balances its budget.

\(^4\) This objective function means that the public sector makes a decision that depends on the preferences of all agents. We could have assumed that policy was set according to the tastes of the median voter, with the paper’s primary qualitative results (diversion, severity effects, and inefficiency) continuing to hold.
This section and the next consider the best-response functions that govern the choices of public and private policing. We will begin by looking at the public sector.

The first-order condition for a interior maximum of $W^*$ with respect to $g$ is

$$\frac{\partial W^*}{\partial g} = \left[ (\partial u/\partial n)(\partial n^*_i/\partial g) + (\partial u/\partial s)(\partial s^*_i/\partial g) \right] - Ic'(g) = 0. \tag{III.3}$$

(III.3) is an instance of the familiar Samuelson condition for the optimal provision of a local public good. The first, bracketed term in (III.3) is the aggregate marginal benefit of public policing, consisting of its effects on the disutility associated with both the level and severity of crime. The second term is the marginal cost. The second-order condition for the government's problem is

$$\frac{\partial^2 W^*}{\partial g^2} = \left[ (\partial u/\partial n)(\partial^2 n^*_i/\partial g^2) + (\partial^2 u/\partial n^2)(\partial n^*_i/\partial g)^2 + 2(\partial^2 u/\partial n \partial s)(\partial n^*_i/\partial g)(\partial s^*_i/\partial g) \right. \\
+ (\partial u/\partial s)(\partial^2 s^*_i/\partial g^2) + (\partial^2 u/\partial s^2)(\partial s^*_i/\partial g)^2 \left. \right] - Ic''(g) < 0. \tag{III.4}$$

Given our earlier assumptions about the curvature of the $v$ and $u$ functions, sufficient, but not necessary, conditions for $\frac{\partial^2 W^*}{\partial g^2} < 0$ are $\frac{\partial^2 n^*_i}{\partial g} > 0$ and $\frac{\partial^2 s^*_i}{\partial g} > 0$. Since $\frac{\partial n^*_i}{\partial g} < 0$ and $\frac{\partial s^*_i}{\partial g} < 0$ these conditions essentially require that the impact of public policing on the level and severity of crime be decreasing at the margin.

The first-order condition (III.3) implicitly defines the government's best response to the vector of private policing levels. By the implicit function theorem, the slope of this best-response function for any is

$$\frac{dg/d\ []}{(\partial^2 W^*/\partial g \partial \[])} = - \frac{(\partial^2 W^*/\partial g \partial \[])}{(\partial^2 W^*/\partial g^2)}, \tag{III.5}$$

where

$$\frac{\partial^2 W^*}{\partial g \partial \[]} = \left[ (\partial u/\partial n)(\partial^2 n^*_i/\partial g \partial \[]) + (\partial^2 u/\partial n^2)(\partial n^*_i/\partial g)(\partial n^*_i/\partial \[]) \right. \\
+ (\partial^2 u/\partial s \partial n)(\partial n^*_i/\partial g)(\partial s^*_i/\partial \[]) + (\partial s^*_i/\partial g)(\partial n^*_i/\partial \[]) \left. \right] + (\partial u/\partial s)(\partial^2 s^*_i/\partial g \partial \[]) \\
+ (\partial^2 u/\partial s^2)(\partial s^*_i/\partial g)(\partial s^*_i/\partial \[]). \tag{III.6}$$
Sufficient, but not necessary, conditions for \( \partial^2 W^*/\partial g \partial g \) \( < 0 \) are \( \partial^2 n_i^*/\partial g \partial g \) \( \geq 0 \) and \( \partial^2 s_i^*/\partial g \partial g \) \( \geq 0 \). We will suppose that these conditions hold. They imply that an increase in either instrument weakly decreases the marginal impact of the other. Then, by (III.5), \( dg/dg \) \(< 0 \) for all \( i \). This means that from the perspective of the government, public and private policing are strategic substitutes.\(^5\)

This analysis highlights one of the important consequences of the mixing of public and private policing: an increase in the level of private policing impacts the incentive to engage in public policing. Because \( \partial^2 u/\partial n^2 \) \( < 0 \), when an increase in private policing reduces the level of crime, the marginal value of a reduction in the level of crime to the public sector is also reduced. A similar effect is at work for severity, when \( \partial^2 v/\partial n \partial s > 0 \). Private policing reduces the level of crime, reducing severity, and in turn decreasing the value of any severity reduction effected by public policing.

This is important because the public sector carries out a different set of crime control activities than does the private sector. An increase in \( g_i \) might involve an increase monitoring activities. A decrease in \( g \) might involve a reduction in police investigations. Monitoring is a general activity, and it reduces the payoff to all crimes, regardless of their severity. In contrast, investigation is a specific activity that penalizes severe crimes more heavily.

Proposition 3 (marginal deterrence): When public and private policing are strategic substitutes, any positive level of private policing decreases marginal deterrence.

Proof: The effect of an increase in \( g \) on the incentive to commit a more severe crime is \( \partial^2 V_i/\partial s \partial g = -F'(g) \). The effect of an increase in \( f \) on the incentive to commit a more severe crime is \( \partial^2 V_i/\partial s \partial f = 0 \). Since \( g \) and \( f \) are strategic substitutes any increase in \( f \) leads to a reduction in \( g \), which means that private policing decreases the additional penalty a criminal incurs from committing a more severe crime. QED

One would suspect that reducing marginal deterrence would lead to an increase in the severity of crime. In order to examine this issue, we must specify the equilibrium determination of the severity and level of crime, which requires that we analyze the private policing choices made by the targets.

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\(^5\) The public sector’s best-response function would be unchanged if instead of assuming equal-share finance, we were to assume that tax shares varied with \( f \). This is a consequence of budget balance and the additive social welfare function that the public sector maximizes.
C. Private policing

Having characterized the public sector's best-response function, we now consider the private sector. In particular, we characterize how the choice of private policing by an individual target varies with the level of private policing chosen by other targets and with the level of public policing. The first-order condition for an interior maximum of $U^*_i$ with respect to $\square$ is

$$\partial U^*_i / \partial \square = (\partial u / \partial n)(\partial n^*_i / \partial \square) + (\partial u / \partial s)(\partial s^*_i / \partial \square) - \square(\square) = 0.$$  

(III.7) says that at the maximizing choice of $\square$, the marginal benefit and marginal cost of private policing are equal. The second-order condition for target i's problem is

$$\partial^2 U^*_i / \partial \square^2 = (\partial u / \partial n)(\partial^2 n^*_i / \partial \square^2) + (\partial^2 u / \partial n^2)(\partial n^*_i / \partial \square) + 2(\partial^2 u / \partial n \partial s)(\partial n^*_i / \partial \square)(\partial s^*_i / \partial \square) + (\partial u / \partial s)(\partial^2 s^*_i / \partial \square^2) - \square''(\square) < 0.$$  

(III.8)

Sufficient, but not necessary, conditions for $\partial^2 U^*_i / \partial \square^2 < 0$ are $\partial^2 n^*_i / \partial \square^2 > 0$ and $\partial^2 s^*_i / \partial \square^2 > 0$. These conditions require that the impact of private policing on the level and severity of crime be decreasing at the margin.

The choice of private policing by any individual target depends on both the private policing of the other targets and on public policing. We will consider the effects of private policing first. By the implicit function theorem, the best response of target i to a change in private policing by target j is given by

$$\partial \square / \partial \square = -(\partial^2 U^*_i / \partial \square^2)(\partial^2 U^*_i / \partial \square^2).$$

From (III.7),

$$\partial^2 U^*_i / \partial \square \partial \square = (\partial u / \partial n)(\partial^2 n^*_i / \partial \square^2) + (\partial^2 u / \partial n^2)(\partial n^*_i / \partial \square)(\partial n^*_i / \partial \square)$$

$$+ (\partial^2 u / \partial n \partial s)(\partial n^*_i / \partial \square)(\partial s^*_i / \partial \square) + (\partial u / \partial s)(\partial^2 s^*_i / \partial \square^2) + (\partial u / \partial s)(\partial^2 s^*_i / \partial \square \partial \square).$$  

(III.9)
It is important to note that in the open version of the model, with $V^*$ fixed, $\partial n^*/\partial g = 0$ by (II.16), which in turn implies $\partial s^*/\partial g = 0$ by (II.20). Substituting these into (III.9) shows that an increase in private policing by target j has no effect on the marginal utility of private policing for target i when the criminal labor market is open. This in turn implies that $\partial g^*/\partial g = 0$. It is difficult to sign (III.9) when the criminal labor market is closed. We will present an example below where the private policing levels are strategic complements.

We now consider target i’s response to the level of public policing. The slope of i’s best-response function is

$$\frac{\partial}{\partial g} = -(\partial^2 U^*_i/\partial g^2)/(\partial^2 U^*_i/\partial g^2),$$

where

$$\partial^2 U^*_i/\partial g^2 = (\partial u/\partial n) (\partial^2 n^*/\partial g^2) + (\partial^2 u/\partial n^2) (\partial n^*/\partial g) (\partial n^*/\partial g)$$

$$+ (\partial^2 u/\partial s^2) (\partial n^*/\partial g) (\partial s^*/\partial g) + (\partial u/\partial s) (\partial^2 s^*/\partial g^2).$$

(III.10)

Sufficient, but not necessary, conditions for $\partial^2 U^*_i/\partial g^2 < 0$ are $\partial^2 n^*/\partial g^2 \geq 0$ and $\partial^2 s^*/\partial g^2 \geq 0$. These are the same conditions that ensure that the best-response function of the public sector is downward sloping (see the discussion after (III.6)). These imply that an individual’s best response to an increase in public policing is to cut back on private policing expenditures: $d\bar{g}/dg < 0$ for all i.

D. Equilibrium

An equilibrium is a vector $(\bar{g}, \bar{g})$ that is simultaneously consistent with the I best-response functions of the targets and the best-response function of the government. Figure 1 illustrates the determination of equilibrium values of $g$ and $\bar{g}$ for some i. These occur at the intersection of the best-response functions, at the point $(g^*, \bar{g}^*)$, a "mixed equilibrium." It should not be surprising given the nonlinear nature of the system that there is not much that can be said about existence in the general case. Later we will address the existence issue constructively.
The equilibrium depicted in Figure 1 is based on the assumption that public and private decisions occur simultaneously. Thus, the public sector’s choice is a best-response to the choices of targets and vice versa. We believe this approach to be the best way to capture the strategic relationships that have governed changes in policing over the last thirty years. As demonstrated in the Introduction, both public and private policing have grown, with private policing growing roughly three times as quickly. In such a fluid environment, an equilibrium should have the Nash property that no agent can benefit from a unilateral deviation.

An alternative approach would be to change the timing of the moves, allowing one party to choose its level of policing first. It seems most natural to entertain the possibility that the public sector is the first mover. This is because the public sector is in some sense incumbent, and its choices may be constrained by prior commitments, such as union contracts or sunk expenditures in policing capital. In this situation, the public sector would choose a level of policing on the best-response curves of the targets. The public sector would thus anticipate the private policing that would be chosen, and relative to a situation without private policing, the public sector would choose a lower level. Diversion and severity externalities would continue to impact the choices of the public sector and the targets. In sum, the results of a three-stage model would be reasonably similar to those of the two-stage model that we consider. One difference that might emerge would be for the public sector to engage in some sort of pre-emption in order to limit private policing. This possibility is not supported by the data: both public and private policing have grown. In a time of growing demand for policing, strategic pre-emption does not seem to occur.6

E. The emergence of a mixed market

It is clear from the evidence reported in the Introduction that the market for policing is becoming increasingly mixed. It is interesting to consider the forces that can lead to mixing in a market that had previously been unmixed. As noted above, it is likely that the target best-response functions define levels of $g_i$ that decrease in the level of $g$. Define $g^0$ to be the level of public policing so that the highest-type target $q_I$ would choose

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6 Since the public sector maximizes welfare, this case is not equivalent to the position of a leader in Stackelberg oligopoly. The public sector would choose $g$ in a way that elicited a response from the targets that would maximize welfare. See Helsley and Strange (2003b) for an analysis of this kind of strategic interaction in a model of a general mixed market (i.e., not a model of crime).
\[ g = 0. \] Thus, if the public sector sets \( g = g^0 \), all targets would choose a zero level of private police. \( g^0 \) must solve

\[
\frac{\partial U_i^*/\partial g}{\partial g} = (\frac{\partial u}{\partial n})(\frac{\partial n_i^*/\partial g}{\partial g}) + (\frac{\partial u}{\partial s})(\frac{\partial s_i^*/\partial g}{\partial g}) - \square(0) = 0.
\]

(III.11)

at \( \square = 0 \). The absence of mixing also requires that \( g^0 \) solve the public sector's maximization problem, with \( g^0 \) and \( \square = 0 \) for all i:

\[
\frac{\partial W^*/\partial g}{\partial g} = \square \frac{((\frac{\partial u}{\partial n})(\frac{\partial n_i^*/\partial g}{\partial g}) + (\frac{\partial u}{\partial s})(\frac{\partial s_i^*/\partial g}{\partial g}))}{Ic'(g^0)} = 0. \quad (III.12)
\]

An unmixed equilibrium is illustrated in Figure 2. It occurs at the horizontal intercept of the public sector's best-response function.

To determine the factors that could lead to mixing, one must identify factors that would increase \( \frac{\partial U_i^*}{\partial g} \) more than they would increase \( \frac{\partial W^*}{\partial g} \). One is a decline in the cost of private policing. Clearly, if \( \square(0) \) falls more rapidly than does \( c'(g^0) \), this would lead a target who was indifferent to private policing to choose a positive level. Also, in the open model, by (II.8) and (II.15), \( \frac{\partial n_i^*}{\partial g} = \frac{(\partial M/\partial g)/(\partial v/\partial n)}{(\partial M/\partial n)/(\partial v/\partial n)} \), while \( \frac{\partial n^*}{\partial g} = (\partial M/\partial g)/(\partial v/\partial n) \), by (II.7) and (II.14). These imply that a relative increase in the productivity of private policing (\( \partial M/\partial g \) rising more than \( \partial M/\partial n \)) would also lead to mixing. This productivity effect is obviously parallel to the cost effect that was just discussed. Figure 2 also illustrates the emergence of a mixed market: if the costs of private policing fell or if private policing were to become more productive, then target best-response functions would shift out, and the market would become mixed, with provision levels \( (g^*, \square^*) \).

The importance of differences in costs between private and public policing suggests that one ought to consider the sources of these differences. There is a growing body of evidence (see the survey by Lopez-de-Silanes et al (1997)) that private provision can reduce the costs of public service provision in general. This may be for economic reasons like a stronger private incentive to control costs or for legal/political/historical reasons like the high unionization rate of the public sector workforce. There are, of course, forces that work in the other direction. Some of these are also economic, for instance the possibility that public provision allows the realization of scale economies.
However, the evidence of substantial scale economies in local public goods provision is weak (Oates (1988)). There are also legal and political factors that may favor public provision. Most importantly, the public sector has the unique ability to exercise force.

These cost differences may arise from changes in technology. Possibly the most important recent change in technology is in communications and information technology. Decreases in the costs of video cameras and alarm systems clearly reduce the cost of private policing. These technological changes are not, however, the only source of the increase in the mixing of policing.

Heterogeneity in targets is another force that can lead to the emergence of mixed markets. To see this, suppose that agents are identical. Now increase heterogeneity such that the public sector choice of $g$ remains the same. In this situation, $g^0$ will no longer discourage private policing since the net disutility of crime, $-(\partial u/\partial n + (\partial u/\partial s)(\partial s/\partial g))$ increases in $\square$. Thus, another force that can give mixed markets is heterogeneity in tastes, a result closely related to the Tiebout hypothesis.

F. Open and closed models of crime

We have characterized the equilibrium of a two-stage game of policing. An important aspect of the crime subgame that has been in the background for much of the analysis is the open or closed nature of the market for crime. In an open model, the targets being analyzed are small, and consequently there are a very large number of potential criminals. These criminals allocate themselves across targets such that each criminal gets the exogenous system-wide criminal utility level. In a closed model, the criminal utility level is endogenous. Any policing carried out by the public sector or by a target will affect this level, and this in turn affects the allocation of criminals across targets.

The strategic interactions are relatively straightforward in the open model case. Each suffers a level of crime that depends on the criminal utility level, which is fixed. This means that the best-response policing level of any target is independent of the private policing choices of other targets. This choice does depend on the level of public policing. In an open model, the prior assumption that $\partial^2 M/\partial g \partial \square \leq 0$ implies that $\partial^2 n^*_j /\partial g \partial \square \leq 0$ and $\partial^2 s^*_j /\partial g \partial \square \geq 0$. By (III.5) and (III.6), these conditions in turn ensure that a target’s optimal level of private policing decreases with the level of public policing. For similar reasons, the level of public policing decreases with the level of private policing.
The closed model is much more complicated. The main differences is that in the closed model the various partial derivatives of \( n_i^* \) and \( s_i^* \) include indirect effects, where the system-wide criminal utility level changes in response to private and public policing (see (II.13) - (II.20)). The most important effect of this is that an increase in private policing in target \( i \) does not simply reduce the crime afflicting target \( i \) itself. It now also reduces the system-wide criminal utility level. This makes the other targets more attractive to criminals, and so the private policing has in effect diverted crime. This, in turn impacts severity when \( \partial^2 v / \partial n \partial s > 0 \). Together, these imply that the private policing choices of various targets are now interdependent.

(III.9) gives the slope of target-\( i \)'s best-response curve with respect to \( g_j \) as:

\[
\frac{\partial^2 U^*_i}{\partial g_i \partial g_j} = (\frac{\partial^2 u}{\partial n^2})(\frac{\partial n_i^*}{\partial g_i})(\frac{\partial n_i^*}{\partial g_j}) + (\frac{\partial u}{\partial n})(\frac{\partial^2 n_i^*}{\partial g_i \partial g_j}) - c
\]  

(III.13)

Suppose that \( u \) is additively separable in \( n \) and \( s \), so that \( \partial^2 u / \partial s \partial n = 0 \). Suppose also that \( v \) is additively separable in \( n \) and \( s \), so that \( \partial^2 v / \partial s \partial n = 0 \). This implies that \( \partial s_i^* / \partial g_i \), \( \partial s_i^* / \partial g_j \), and \( \partial^2 s_i^* / \partial g_i \partial g_j \) all equal zero. In this case,

\[
\frac{\partial^2 U^*_i}{\partial g_i \partial g_j} = (\frac{\partial^2 u_i^*}{\partial n^2})(\frac{\partial n_i^*}{\partial g_i})(\frac{\partial n_i^*}{\partial g_j}) + (\frac{\partial u_i^*}{\partial n})(\frac{\partial^2 n_i^*}{\partial g_i \partial g_j}) - c
\]  

(III.14)

The first and third terms of (III.14) are positive. The second is ambiguous. If we also assume that \( v \) is linear in \( n \) so \( \partial^2 v / \partial n^2 = 0 \), then \( \partial n_i^* / \partial g_i \partial g_j = 0 \) as well. In this case, \( g_i \) and \( g_j \) are strategic complements. Because target-\( i \)'s marginal disutility from crime is increasing, when target \( j \) increases its level of private policing, target \( i \) will respond in the same way. Clearly, the strategic interactions in the closed model are significantly more complicated than in the open model. There are two issues. First, the targets affect each other. Second, the best-responses of the public sector and targets depend on indirect effects operating through the system-wide criminal utility level.

G. Example

In this section, we present and interpret a numerical example. We have three reasons for doing this. First, the example serves as a constructive proof of the existence
of equilibrium. Second, the example allows us to illustrate the general equilibrium effects of mixing on crime and policing in a closed model. This is a global comparison between a mixed market and one where all policing is public, so it is not possible to say much in the general case. It is important, however, to illustrate the range of effects on crime and severity from moving to a mixed market, and the example does that.

The example is based on the following specification: $$V_i = \left[ b_0 - b_1 n + \ln(a_1 s) \right] - M_1 (g + \bar{g}) - F_1 g s, U_i = \left[ n^2 + s^2 \right] - \bar{g} - c g,$$ and $$N^S = q_0 + q_1 V.$$ All parameters are non-negative. There are two targets ($I = 2$). In this specification, we are able to obtain closed form analytical solutions for $$n^*_i, s^*_i,$$ and $$V^*.$$ We are also able to present algebraic expressions defining the best-response functions and the equilibrium levels of $$g$$ and $$\bar{g}.$$ Unfortunately, despite the simplifications, these solutions are unwieldy. Consequently, we will present the example numerically, using the following as base case parameters: $$a_1 = 1, b_0 = 1000, b_1 = 10, g_1 = 1, F_1 = 1, M_1 = 1, q_0 = 20, q_1 = 1, \bar{g} = 10, and c = 10.$$ Using the base case parameters, we begin by maximizing $$V_i$$ over $$s,$$ giving $$s^*_i = \frac{q_i}{g}.$$ Substituting this into $$V_i$$ and solving gives $$n^*_i = \frac{1}{120} [10190 - 10 g + 10 \ln(1/g)].$$ It is not possible to go further without specifying target types. We will begin with the symmetric case. Setting $$q_1 = q_2 = 1,$$ aggregating to obtain aggregate demand for crime, setting this equal to the supply of crime, and solving for the utility level gives: $$V = \frac{1}{12} [1798 - 2 g - 11 \bar{g} + 110 \ln(1/g)].$$ This in turn gives $$n^*_1 = \frac{1}{120} [10190 - 10 g - 11 \bar{g} + 10 \ln(1/g)]$$ and $$n^*_2 = \frac{1}{120} [10190 - 10 g - \bar{g} + 10 \ln(1/g)],$$ with $$s^*_1 = s^*_2 = \frac{1}{g}.$$ Maximizing over target utility gives the best-response functions for $$\bar{g}:$$

$$\bar{g}(\bar{g}, g) = \frac{1}{121} [40090 - 110 g + 11 \bar{g} + 110 \ln(1/g)].$$ (III.15)

The best-response function $$\bar{g}(\bar{g}, g)$$ is symmetrical. The best-response level of $$g$$ is implicitly defined by:

$$-288 + 2 g^4 + g^2 (-2038 + \bar{g} + \bar{g}) + g^3 (-596 + \bar{g} + \bar{g}) + 2 g^2 (1 + g) + \ln(1/g) = 0.$$ (III.16)

The solutions for policing instruments for the symmetric case, $$\bar{g} = \bar{g}_1 = 1,$$ and for asymmetric cases, $$\bar{g} = 0.9$$ and $$\bar{g}_1 = 1.1,$$ and $$\bar{g} = 0.8$$ and $$\bar{g}_1 = 1.2,$$ are given in Table 1. The first aspect of the table to focus on is the solution for policing instruments. Not surprisingly, in the mixed equilibrium the $$\bar{g}$$ choices become increasingly different as the targets become more heterogeneous. In the case where $$\bar{g}_1 = 0.8$$ and $$\bar{g}_1 = 1.2,$$ so target 2 is half again as attractive to criminals as is target 1, target 2 sets a level of private
policing that is more than seventy five percent larger. The remainder of the table solves for equilibrium levels of crime, severity, and target utility.

Table 2 reports parallel solutions for an unmixed market, where $g_1 = g_2 = 0$. The comparison to the mixed market solution is consistent with the analysis thus far: the mixed market involves less public policing and more private policing. In the symmetric case, the level of crime falls for both targets, but the level of severity rises because of the change in the technology of policing. In the asymmetric cases, the level of crime always falls for the high-type target. The effect on the low-type target is ambiguous. With more asymmetry, the difference between $g_1$ and $g_2$ is greater. In this case, the level of crime for the low-type target will rise. If the difference between $g_1$ and $g_2$ is not too great, then the level of crime for the low-type target may fall. The severity of crime rises for both targets in all cases as a result of the decrease in public policing and the associated loss of marginal deterrence.

We will return to this example to consider the welfare effects of mixed markets. Prior to doing so, it is necessary to specify efficient program for public and private policing.

IV. Optimal mixing

In the previous section we examined the provision of public and private policing in the context of a simple game played by a government, a number of private targets and a population of potential criminals. We demonstrated that the Nash equilibrium of that game sometimes involves mixing -- that is, the simultaneous public and private provision of crime control. This section considers the efficiency of such a mixed equilibrium.

We will establish two normative results. First, the mixed equilibrium is generally inefficient in the sense that the equilibrium level of private policing is excessive. Second, private policing is not necessarily beneficial to the targets who choose to provide it: the mixed equilibrium can be Pareto inferior to the unmixed equilibrium in which there are no private attempts at crime control.

A. Welfare maximization

To establish these results we will focus on the levels of public and private policing that maximize the welfare of targets, anticipating how the crime control instruments impact the level and severity of crime, as discussed in Section II. Choosing welfare maximization as our objective has the obvious advantage of making the optimum
program directly comparable to the government's maximization program in the equilibrium. Since we have assumed transferable utility, the welfare maximum is Pareto efficient.

The planner's problem is to choose $g$ and $\bar{g}$ to maximize $W^*$, as given by (III.2). The first-order condition that characterizes an optimal choice of $g$ is, of course, identical to (III.3), the first-order condition that characterizes the government's choice of $g$ in the equilibrium program. The first-order condition that characterizes an optimal choice of $\bar{g}$ is

$$\frac{\partial W^*}{\partial g_i} = \left( \frac{\partial u}{\partial n} \right) \frac{\partial n^*_i}{\partial g_i} + \left( \frac{\partial u}{\partial s} \right) \frac{\partial s^*_i}{\partial g_i} - c'(g_i) + \{S_{j \neq i} \left[ \left( \frac{\partial u}{\partial n} \right) \frac{\partial n^*_j}{\partial g_i} + \left( \frac{\partial u}{\partial s} \right) \frac{\partial s^*_j}{\partial g_i} \right] \} = 0. \quad (IV.1)$$

The first three terms in (IV.1) are familiar from (III.7): they equal $\frac{\partial U^*}{\partial g_i}$. The other, bracketed term is the sum of the external effects of $\bar{g}$ on the level and severity of crime for all other targets. (II.16) and (II.20), together with our earlier assumptions, imply that every term in the bracketed sum in (IV.1) is non-positive. If the criminal labor market is closed, and $\frac{\partial^2 v}{\partial s \partial n} > 0$, then every term in the bracketed sum in (IV.1) is strictly negative. Thus, evaluating (IV.1) at the equilibrium values of $\bar{g}$ and $g$ (using by (III.7)), we have

$$\frac{\partial W^*}{\partial g_i} (\bar{g}, g^E) = \{S_{j \neq i} \left[ \left( \frac{\partial u}{\partial n} \right) \frac{\partial n^*_j}{\partial g_i} + \left( \frac{\partial u}{\partial s} \right) \frac{\partial s^*_j}{\partial g_i} \right] \} < 0. \quad (IV.2)$$

This leads to the following result:

Proposition 4 (efficiency): Suppose the criminal labor market is closed. Then the level of private policing is excessive in equilibrium.

Proof: Under these conditions $\frac{\partial W^*}{\partial g_i} (\bar{g}, g^E) < 0$. Then the second-order conditions imply that reducing $\bar{g}$ from its equilibrium level would cause welfare to rise. QED

The source of the difference between the equilibrium and the optimum is the externality associated with private policing. As established earlier, when the criminal labor market is closed and severity increases with the level of crime, an increase by one target increases both the level and severity of crime for all others. Thus, at the margin, private policing is excessive.
In Section III, we noted that if the criminal labor market is open, private policing has no diversionary effects. In this case, \( \partial n^* / \partial g = 0 \), \( i \neq j \). Thus, an obvious corollary to Proposition 4 is that if the criminal labor market is open, then the equilibrium from Section III is efficient.

**B. Welfare effects**

Having established that the mixed equilibrium fails to maximize welfare, we now consider the global welfare effects of mixing. This involves comparing the mixed and unmixed equilibria. However, making global comparisons in this general equilibrium setting is very difficult. Our strongest results will be for a polar case. This polar case is based on three additional assumptions. The first is that all targets are homogeneous: \( q_i = q \) for all \( i \). With homogeneous targets, the equilibrium and optimum allocations will obviously be symmetric, and we will drop the subscript \( i \) to indicate that we are focusing on a symmetric outcome. The second assumption is that public and private policing are perfect substitutes in the \( M \) function: \( M(g, \overline{g}) = M \cdot (g + \overline{g}) \), where \( M \) is a positive constant. The third assumption is that the marginal per capita costs of public and private policing are equal and constant. The cost of the policing level \( g + \overline{g} \) will be \( C \cdot (g + \overline{g}) \), where \( C \) is a positive constant.

Working back through some of our earlier analysis, (II.7) and (II.8) imply that in the polar case,

\[
\partial n / \partial g - \partial n / \partial \overline{g} = (F'(g)s) / (\partial v / \partial n) < 0.
\]

(IV.3)

Thus, although \( g \) and \( \overline{g} \) are perfect substitutes through the \( M \) function, \( g \) has a larger marginal impact on crime through its role as an instrument of marginal deterrence. Using (IV.3), (II.11) and (II.12) imply

\[
\partial V^* / \partial g - \partial V^* / \partial \overline{g} = - [ (F'(g)s) / (\partial v / \partial n) ] (N^D - N^S) < 0,
\]

(IV.4)

so that \( g \) also has a larger impact on the market clearing criminal payoff level \( V^* \). Using (IV.4), (II.14) and (II.15) imply

\[
\partial n^* / \partial g - \partial n^* / \partial \overline{g} = - [ (F'(g)s) / (\partial v / \partial n) ] [N^D / (N^D - N^S)] < 0.
\]

(IV.5)

Using (IV.5), (II.18) and (II.19) imply
\[ \frac{\partial s^*/\partial g - \partial s^*/\partial n}{\partial s/\partial n} = \frac{\partial s/\partial g - (\partial s/\partial n)(F'(g)s)/(\partial v/\partial n))}[N^s/(N^D - N^s)] < 0. \quad \text{(IV.6)} \]

The first-order conditions for welfare maximization in this case are

\[ \frac{\partial W^*/\partial g}{\partial u/\partial n}(\partial n^*/\partial g) + (\partial u/\partial s)(\partial s^*/\partial g) - C = 0, \quad \text{(IV.7)} \]

and

\[ \frac{\partial W^*/\partial g}{\partial u/\partial n}(\partial n^*/\partial g) + (\partial u/\partial s)(\partial s^*/\partial g) - C = 0. \quad \text{(IV.8)} \]

Using (IV.5) and (IV.6), (IV.8) implies

\[ (\partial u/\partial n)(\partial n^*/\partial g) + (\partial u/\partial s)(\partial s^*/\partial g) - C = (\partial u/\partial n)(F'(g)s)/(\partial v/\partial n))][N^s/(N^D - N^s)] - (\partial u/\partial s)(\partial s/\partial g + (\partial s/\partial n)(F'(g)s)/(\partial v/\partial n))[-N^s/(N^D - N^s)] < 0. \quad \text{(IV.9)} \]

Thus, in this polar case, \( \frac{\partial W^*/\partial g}{\partial u/\partial n}(\partial n^*/\partial g) + (\partial u/\partial s)(\partial s^*/\partial g) - C < 0. \)

This analysis may be summarized as follows:

Proposition 5 (rat race): Suppose targets are homogeneous, that private and public policing are perfect substitutes, and that the marginal costs of private and public policing are equal and constant. Then, if \( g > 0 \) in a mixed equilibrium, all targets are worse off in the mixed equilibrium than in the unmixed equilibrium.

Proof: Under these conditions, \( \frac{\partial W^*/\partial g}{\partial u/\partial n}(\partial n^*/\partial g) + (\partial u/\partial s)(\partial s^*/\partial g) - C < 0. \) Then the second-order conditions imply that the optimal value of \( g \) is the corner solution \( g = 0. \) QED

Proposition 5 shows that when targets are identical, and there is no cost advantage to private policing, then a positive amount of private policing makes all targets worse-off. The targets choose levels of private policing under the usual Nash assumption regarding the policing levels of the other targets. Under this assumption, they are correct in believing that private policing reduces their own crime and severity levels. Private policing does this in part by diverting crime to other targets. The problem is that the other targets are carrying out the same calculations, and the crime diverted by one's own
private policing is returned by the private policing of others. For instance, it may be a best-response for one building to hire a doorman or bar its windows, and for neighbors to do likewise, with the ultimate effect being that all are worse-off. Of course, whether this effect is common or rare is an empirical question.

The one issue that remains is to show that targets may set positive levels of private policing when they all end up worse off. Under the assumption of identical and linear costs, the first-order condition for \( g \) given by (III.7) becomes

\[
\frac{\partial U^*}{\partial g} = (\partial u/\partial n)(\partial n^*/\partial g) + (\partial u/\partial s)(\partial s^*/\partial g) - C = 0. \tag{IV.10}
\]

The key difference between this and the condition that governs the optimal choice of \( g \) in the symmetric case is in the first term, which in the optimum program is \((\partial u/\partial n)(\partial n^*/\partial g)\). The expression \( \partial n^*/\partial g \) incorporates the symmetry assumption that all \( I \) targets will set the same \( g \). The expression \((\partial n^*/\partial g)\) assumes otherwise, with the other \( I - 1 \) targets not changing their private policing levels. Thus, the latter is larger in absolute value. This means that even when it is not optimal to choose a positive level of private policing \( (\partial W^*/\partial g < 0 \text{ at } g = 0) \), individual targets may find their interests better served with \( g > 0 \). It is important to point out that this is not guaranteed. If severity were very costly to targets, for example, then the public sector would set \( g \) very high. It is possible that this would discourage private policing in the mixed equilibrium.

To illustrate the possibility of a rat race, return to the example discussed in Section III. The final two columns of Tables 1 and 2 give the utility levels of the two targets in the mixed and unmixed equilibria, respectively. In the case where targets are very heterogeneous \( (q_1 = 0.8 \text{ and } q_2 = 1.2) \), the targets are better-off in aggregate in the mixed equilibrium. However, this is a situation where the high-type target has gained at the expense of the low-type target. The difference in target-1 utility (mixed minus unmixed) is -158.08, while the difference for target-2 is 224.79. The decentralization that is associated with mixing allows the high-type target to choose a policing level that better accords with tastes. Target-2 is made better off despite increases in both the crime level and severity. Target-1, on the other hand, is worse-off. This is not the only possibility.

The case where the targets are moderately heterogeneous \( (q_1 = 0.9 \text{ and } q_2 = 1.1) \) gives a difference in utility from mixing of -128.007 for target-1 and 49.3333 for target-2. Thus, total target welfare falls, with target-1 is worse-off and target-2 only slightly better-off.

---

7 This bears some resemblance to results in the literature on mixed oligopoly where adding a public participant to a market does not necessarily improve welfare. See De Fraja and Delbono (1990)
The case where targets are identical ($q_1 = 1$ and $q_2 = 1$) gives a rat race, as in Proposition 5. Moving to a mixed market reduces target utility by -63.5681. Thus, the example describes a situation where private policing can make all targets worse-off. This result is reminiscent of results in the literature on rent-seeking and on the endogenous determination of property rights. In rent-seeking models, agents compete against each other for governmental favors. The competition consumes resources and, because it is entirely concerned with distribution, it produces nothing. It is possible that all participants be worse-off, at least from an ex ante perspective. In models of the endogenous determination of property rights, agents expend resources protecting their own property rights and attempting to expropriate other agents. Again, it is possible for all agents to be worse-off (Skapedras (1992) and Grossman and Kim (1995)).

V. Conclusion

This paper has focused on mixed markets and crime. It is natural to consider policy implications. One important result is that private policing has externalities, resulting in the diversion of crime to unprotected targets and also increasing severity. Thus, it is likely that private policing is excessive. If the public sector were to attempt to control private policing, how might it go about this? One way would be to impose a Pigovian tax. In most cities, home alarm systems are taxed, but the tax is typically described as a charge for police call-outs rather than in any way reflecting the costs of diversion. Furthermore, most private policing activities (i.e., hiring guards, building fortification, or carrying out video surveillance) are not. Thus, there seems to be room for improvement in this area.

This analysis has implications for other mixed markets. One could argue that the most important of these are education and transportation. In what sense is the analysis of these markets informed by this paper's conclusions? There are a number of potential parallels. First, there are likely to be political incentive externalities, with an increase in private provision to lead to a decrease in public provision. Or vice versa. Second, there are likely to be different instruments available to public and private providers. In our paper, the public sector is modeled as having more instruments than the private sector. The opposite is also possible. In the case of education, the private sector can teach religion, while the public sector sometimes cannot. A move to a mixed market, therefore, will change the technology of education provision. Third, the rat race result could extend to a model of transportation, with private automobile purchases reducing public mass transit infrastructure, which in turn increases the incentives of agents to buy cars.
References


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Note: this table solves for mixed market equilibrium with $V_i = \bar{q} [b_0 - b_1 n + \ln(a_1 s)] - M_i (g + \bar{q}) - F_i g_s$, $U_i = \underline{q} (n^2 + s^2) - \underline{q} - c g$, and $N^S = q_0 + q_i V$. Base case parameters are $a_i = 1, b_0 = 1000, b_1 = 10, M_i = 1, F_i = 1, q_0 = 20, q_i = 1, \bar{q} = 10$, and $c = 10$.

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Note: this table solves for mixed market equilibrium with $V_i = \bar{q} [b_0 - b_1 n + \ln(a_1 s)] - M_i (g + \bar{q}) - F_i g_s$, $U_i = \underline{q} (n^2 + s^2) - \underline{q} - c g$, and $N^S = q_0 + q_i V$. Base case parameters are $a_i = 1, b_0 = 1000, b_1 = 10, M_i = 1, F_i = 1, q_0 = 20, q_i = 1, \bar{q} = 10$, and $c = 10$. 

Figure 1. Equilibrium
Note: the lower $g_0(g_0, \bar{g}_0)$ best-response function corresponds to a case where targets do not have an incentive to provide a positive level of policing. The higher $g^*_0(g_0, \bar{g}_0)$ best-response function -- which could be reached if private policing costs were to fall -- corresponds to the emergence of a mixed market.